Sèrie 2

Exercici 1

Q1 b

Q2 c

Q3 d

Q4 b

Q5 a

Exercici 2

a)

$$U_{\text{an}} = \frac{U}{\sqrt{3}} = \frac{400}{\sqrt{3}} = 230{,}94 \text{ V}$$

b)

$$I_{\text{a}} = \frac{U_{\text{an}}}{Z} = \frac{U_{\text{an}}}{|\underline{Z}|} = \frac{U_{\text{an}}}{\sqrt{R^2 + X^2}} = \frac{230{,}94}{\sqrt{12^2 + 7^2}} = 16{,}623 \text{ A}$$

c)

$$P = 3\,R\,I_{\text{a}}^2 = 3 \cdot 12 \cdot 16{,}623^2 = 9{,}948 \text{ kW}$$

$$Q = 3\,X\,I_{\text{a}}^2 = 3 \cdot 7 \cdot 16{,}623^2 = 5{,}803 \text{ kvar}$$

Alternativament,

$$\varphi = \tan^{-1}\left(\frac{X}{R}\right) = \tan^{-1}\left(\frac{7}{12}\right) = 30{,}2564°$$

$$P = \sqrt{3}\,U\,I_{\text{a}}\,\cos\varphi = \sqrt{3} \cdot 400 \cdot 16{,}623 \cdot \cos(30{,}2564°) = 9{,}948 \text{ kW}$$

$$Q = \sqrt{3}\,U\,I_{\text{a}}\,\sin\varphi = \sqrt{3} \cdot 400 \cdot 16{,}623 \cdot \sin(30{,}2564°) = 5{,}803 \text{ kvar}$$

Alternativament, en cas que se sàpiga operar amb números complexos,

$$\underline{S} = P + j\,Q = \frac{U^2}{\underline{Z}^*} = \frac{400^2}{12 - j\,7} = \underbrace{9{,}948}_{9{,}948 \text{ kW}} + j\,\underbrace{5{,}803}_{5{,}803 \text{ kvar}} \text{ kVA}$$

d)

$$I = 0\ \text{A}$$

Exercici 3

a)

$$\omega_\text{N} = \frac{P_\text{N}}{\Gamma_\text{N}} = \frac{7500}{50} = 150\ \frac{\text{rad}}{\text{s}} \qquad \rightarrow \qquad n_N = \omega_\text{N}\ \frac{60}{2\,\pi} = 150\ \frac{60}{2\,\pi} = 1432{,}4\ \text{min}^{-1}$$

b)

$$\eta(\%) = 100\ \frac{P_\text{N}}{\sqrt{3}\ U_\text{N}\ I_\text{N}\ \cos\varphi_\text{N}} = 100\,\frac{7500}{\sqrt{3}\cdot 400\cdot 14{,}5\cdot 0{,}82} = 91{,}05\ \%$$

c)

$$\omega_\text{s} = \frac{\omega}{p} = \frac{2\,\pi\,f_\text{N}}{p} = \frac{2\,\pi\,50}{2} = 157{,}08\ \frac{\text{rad}}{\text{s}}$$

$$s_\text{N}(\%) = 100\ \frac{\omega_\text{s} - \omega_\text{N}}{\omega_\text{s}} = 100\ \frac{157{,}08 - 150}{157{,}08} = 4{,}51\ \%$$

d)

$U_\text{triangle} = \frac{U_\text{estrella}}{\sqrt{3}} = \frac{400}{\sqrt{3}} = 230{,}94\ \text{V}$ Si indiquen 230 V, també és correcte.

$$I_\text{triangle} = \sqrt{3}\ I_\text{estrella} = \sqrt{3}\cdot 14{,}5 = 25{,}11\ \text{A}$$

e)

$$I = I_\text{N} = 14{,}5\ \text{A}$$

Exercici 4

a)

$$I_1 = \frac{U_1}{R_1 + R_3} = \frac{12}{5+5} = 1{,}2\ \text{A}$$

$$P_{\text{R1}} = R_1\, I_1^2 = 5 \cdot 1{,}2^2 = 7{,}2\ \text{W}$$

b)

$$P_{\text{R1}} = 0\ \text{W} \quad \rightarrow \quad I_1 = 0\ \text{A} \quad \rightarrow \quad U_{\text{R3}} = U_1 = 12\ \text{V}$$

$$I_{\text{R3}} = \frac{U_{\text{R3}}}{R_3} = \frac{12}{5} = \frac{U_1 + U_2}{R_2 + R_3} = \frac{12+12}{R_2+5} = \frac{24}{R_2+5} = 2{,}4\ \text{A}$$

$$2{,}4\,(R_2+5) = 24 \quad \rightarrow \quad R_2 = \frac{24}{2{,}4} - 5 = 5\ \Omega$$

c)

$$P_{\text{U1}} = U_1\, I_{\text{R3}} = 12 \cdot 2{,}4 = 28{,}8\ \text{W}$$

d)

$$P_{\text{R3}} = R_3\, I_{\text{R3}}^2 = \frac{U_{\text{R3}}^2}{R_3} = 5 \cdot 2{,}4^2 = \frac{12^2}{5} = 28{,}8\ \text{W}$$

Exercici 5

a)

$$X_{\mathrm{L}} = \omega\, L = 2\,\pi\, f\, L = 2\,\pi\, 50 \cdot 100 \cdot 10^{-3} = 31{,}416\ \Omega$$

$$X_{\mathrm{C}} = \frac{1}{\omega\, C} = \frac{1}{2\,\pi\, f\, C} = \frac{1}{2\,\pi\, 50 \cdot 65 \cdot 10^{-6}} = 48{,}971\ \Omega$$

$$A = I = \frac{U}{Z} = \frac{U}{\sqrt{R^2 + (X_{\mathrm{L}} - X_{\mathrm{C}})^2}} = \frac{100}{\sqrt{12^2 + (31{,}416 - 48{,}971)^2}} = 4{,}703\ \mathrm{A}$$

b)

$$W = P_{\mathrm{R}} = R\, I^2 = 12 \cdot 4{,}703^2 = 265{,}42\ \mathrm{W}$$

c)

Com $X_{\mathrm{L}} < X_{\mathrm{C}}$, el factor de potència és **capacitiu**.

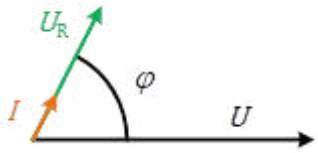
$$fdp = \cos\varphi = \frac{R}{Z} = \frac{R}{Z} = \frac{12}{\sqrt{12^2 + (31{,}416 - 48{,}971)^2}} = 0{,}5643$$

Alternativament,

$$\varphi = \tan^{-1}\left(\frac{X_{\mathrm{L}} - X_{\mathrm{C}}}{R}\right) = \tan^{-1}\left(\frac{31{,}416 - 48{,}971}{12}\right) = -55{,}645°$$

$$fdp = \cos\varphi = \cos(-55{,}645°) = 0{,}5643$$

d)



Tot i que no es demanava a l'enunciat del problema, els valors corresponents són:

$$U = 100\ \mathrm{V} \qquad I = 4{,}703\ A \qquad \varphi = 55{,}645° \qquad U_{\mathrm{R}} = R\, I = 56{,}436\ \mathrm{V}$$

Exercici 6

a)

$$\Delta U_{\text{màx}} = \frac{3}{100}\, U_{\text{N Línia}} = \frac{3}{100}\, 230 = 6{,}9 \text{ V}$$

Amb un 3% de caiguda de tensió, la tensió en borns del transformador és:

$$U_{\text{trafo}} = U_{\text{N Línia}} - \Delta U_{\text{màx}} = 230 - 6{,}9 = 223{,}1 \text{ V}$$

El corrent que circula pel primari del transformador:

$$R_{\text{Eq.}} = \frac{U^2_{\text{primari N}}}{P_{\text{Lluminàries}}} = \frac{230^2}{10 \cdot 100} = 52{,}9\ \Omega$$

Alternativament,

$$R_{1\ \text{Lluminària}} = \frac{U^2_{\text{N Lluminària}}}{P_{\text{Lluminària}}} = \frac{6^2}{100} = 0{,}36\ \Omega$$

$$R_{\text{Eq. Secundari}} = \frac{1}{\sum_{i=1}^{10} \frac{1}{R_i}} = \frac{1}{10\ \frac{1}{0{,}36}} = 0{,}036\ \Omega$$

$$R_{\text{Eq.}} = R_{\text{Eq. Secundari}}\, r_{\text{t}}^2 = 0{,}036 \left(\frac{230}{6}\right)^2 = 52{,}9\ \Omega$$

$$I = \frac{U_{\text{trafo}}}{R_{\text{Eq.}}} = \frac{223{,}1}{52{,}9} = 4{,}217 \text{ A}$$

$$2\, R_{\text{màx}} = \frac{\Delta U_{\text{màx}}}{I} \quad \rightarrow \quad R_{\text{màx}} = \frac{\Delta U_{\text{màx}}}{2\, I} = \frac{6{,}9}{2 \cdot 4{,}217} = 0{,}8181\ \Omega$$

$$R_{\text{màx}} = \rho\, \frac{L}{S_{\text{mín}}} \quad \rightarrow \quad S_{\text{mín}} = \frac{\rho\, L}{R_{\text{màx}}} = \frac{0{,}01786 \cdot 10^{-6} \cdot 150}{0{,}8181} = 3{,}275 \cdot 10^{-6}\ \text{m}^2 = 3{,}275\ \text{mm}^2$$

b)

La secció escollida, és, doncs, $S = 4\ \text{mm}^2$

c)

$$R_{\text{Línia}} = \rho \frac{2\,L}{S} = \frac{0{,}01786 \cdot 10^{-6} \cdot 2 \cdot 150}{4 \cdot 10^{-6}} = 1{,}3395\ \Omega$$

$$I = \frac{U_{\text{N Línia}}}{R_{\text{Total}}} = \frac{230}{1{,}3395 + 52{,}9} = 4{,}24\ \text{A}$$

$$U_{\text{Lluminàries}} = \frac{U_{\text{trafo}}}{r_{\text{t}}} = \frac{R_{\text{Eq.}}\,I}{r_{\text{t}}} = \frac{52{,}9 \cdot 4{,}24}{\frac{230}{6}} = 5{,}851\ \text{V}$$

SÈRIE 5

Exercici 1

Q1 b

Q2 a

Q3 d

Q4 b

Q5 b

Exercici 2

a)

$$R_{\mathrm{EqO}} = \frac{(R+3\,R)\,(2\,R+4\,R)}{(R+3\,R)+(2\,R+4\,R)} = \frac{24}{10}\,R \qquad \rightarrow \qquad U = R_{\mathrm{EqO}}\,I_{\mathrm{O}} = 2{,}4\,R\,3 = 7{,}2\,R$$

$$R_{\mathrm{EqT}} = \frac{(R)\,(2\,R)}{(R)+(2\,R)} + \frac{(3\,R)\,(4\,R)}{(3\,R)+(4\,R)} = \left(\frac{2}{3}+\frac{12}{7}\right) R = \frac{50}{21}\,R \qquad P_{\mathrm{T}} = \frac{U^2}{R_{\mathrm{EqT}}}$$

$$413{,}68 = \frac{(7{,}2\,R)^2}{\frac{50}{21}\,R} \qquad \rightarrow \qquad R = \frac{413{,}68\,\frac{50}{21}}{7{,}2^2} = 19\ \Omega$$

$$U = 7{,}2\,R = 7{,}2 \cdot 19 = 136{,}8\ \mathrm{V}$$

b)

$$I_{\mathrm{SW}} = I_{\mathrm{R}} - I_{3\mathrm{R}}$$

$$I_{\mathrm{R}} = \frac{U_{\mathrm{R}}}{R} = \frac{\dfrac{\dfrac{(R)\,(2\,R)}{(R)+(2\,R)}}{\dfrac{(R)\,(2\,R)}{(R)+(2\,R)}+\dfrac{(3\,R)\,(4\,R)}{(3\,R)+(4\,R)}}\,U}{R} = \frac{\frac{2}{3}}{\frac{50}{21}}\,136{,}8}{19} = 2{,}016\ \mathrm{A}$$

$$I_{3\mathrm{R}} = \frac{U_{3\mathrm{R}}}{3R} = \frac{\dfrac{\dfrac{(3\,R)\,(4\,R)}{(3\,R)+(4\,R)}}{\dfrac{(R)\,(2\,R)}{(R)+(2\,R)}+\dfrac{(3\,R)\,(4\,R)}{(3\,R)+(4\,R)}}\,U}{3R} = \frac{\frac{12}{7}}{\frac{50}{21}}\,136{,}8}{3 \cdot 19} = 1{,}728\ \mathrm{A}$$

$$I_{\mathrm{SW}} = I_{\mathrm{R}} - I_{3\mathrm{R}} = 2{,}016 - 1{,}728 = 0{,}288\ \mathrm{A}$$

c)

$$P_O = U\, I_O = 136{,}8 \cdot 3 = 410{,}4\ \text{W}$$

Exercici 3

a)

$$W_1 = R\, A_1^2 \quad \rightarrow \quad R = \frac{W_1}{A_1^2} = \frac{400}{4^2} = 25\ \Omega$$

b)

$$A_1 = \frac{\frac{U}{\sqrt{3}}}{\sqrt{R^2 + X_L^2}} \quad \rightarrow \quad R^2 + X_L^2 = \left(\frac{\frac{U}{\sqrt{3}}}{A_1}\right)^2 \quad \rightarrow \quad X_L = \sqrt{\left(\frac{\frac{U}{\sqrt{3}}}{A_1}\right)^2 - R^2}$$

$$X_L = \sqrt{\left(\frac{\frac{400}{\sqrt{3}}}{4}\right)^2 - 25^2} = 52{,}04\ \Omega \qquad L = \frac{X_L}{\omega} = \frac{X_L}{2\,\pi\, f} = \frac{52{,}04}{2\,\pi\, 50} = 165{,}6\ \text{mH}$$

c)

$$V_1 = X_L\, A_1 = 52{,}04 \cdot 4 = 208{,}16\ \text{V}$$

d)

$$Q = X_L\, A_1^2 = 52{,}04 \cdot 4^2 = 832{,}64\ \text{var}$$

$$X_C = \frac{U^2}{Q} = \frac{400^2}{832{,}64} = 192{,}16\ \Omega \qquad C = \frac{1}{\omega\, X_C} = \frac{1}{2\,\pi\, f\, X_C} = \frac{1}{2\,\pi\, 50 \cdot 192{,}16} = 16{,}56\ \mu\text{F}$$

Exercici 4

a)

$$f = \frac{1}{T} = \frac{1}{4\ \text{div} \cdot 10\ \frac{\text{ms}}{\text{div}} \cdot \frac{1\ \text{s}}{1000\ \text{ms}}} = 25\ \text{Hz}$$

b)

Quan condueixen D_2 i D_4:

$$I_{G_1\,\text{Min}} = I_{R_1\,\text{Min}} = I_{R_2\,\text{Min}} = I_{R_3} = \frac{-1\ \text{div} \cdot 5\ \frac{\text{V}}{\text{div}}}{R_1} = \frac{-5}{5} = -1\ \text{A}$$

$$I_{G_1\,\text{Min}} = \frac{U_{G_1\,\text{Min}}}{R_1 + R_2 + R_3} \quad \rightarrow \quad R_3 = \frac{U_{G_1\,\text{Min}}}{I_{G_1\,\text{Min}}} - R_1 - R_2 = \frac{-100}{-1} - 5 - 75 = 20\ \Omega$$

c)

Quan condueixen D_1 i D_3:

$$I_{G_1\,\text{Max}} = I_{R_1\,\text{Max}} = I_{R_4} = \frac{2{,}5\ \text{div} \cdot 5\ \frac{\text{V}}{\text{div}}}{R_1} = \frac{12{,}5}{5} = 2{,}5\ \text{A}$$

Alternativament,

$$I_{G_1\,\text{Max}} = I_{R_1\,\text{Max}} = I_{R_4} = \frac{U_{G_1\,\text{Max}} - U_2}{R_1 + R_4} = \frac{100 - 25}{5 + 25} = 2{,}5\ \text{A}$$

$$\overline{P}_{\text{Total R}} = \overline{P}_{\text{R1}} + \overline{P}_{\text{R2}} + \overline{P}_{\text{R3}} + \overline{P}_{\text{R4}} + \overline{P}_{\text{R5}}$$

$$\overline{P}_{\text{R1}} = R_1\, I^2_{G_1\,\text{Max}}\, \frac{2{,}5\ \text{div}}{4\ \text{div}} + R_1\, I^2_{G_1\,\text{Min}}\, \frac{1{,}5\ \text{div}}{4\ \text{div}} = R_1 \left(I^2_{G_1\,\text{Max}}\, \frac{2{,}5}{4} + I^2_{G_1\,\text{Min}}\, \frac{1{,}5}{4} \right)$$

$$\overline{P}_{\text{R1}} = 5 \left(2{,}5^2\, \frac{2{,}5}{4} + 1^2\, \frac{1{,}5}{4} \right) = 21{,}406\ \text{W}$$

$$\overline{P}_{\text{R2}} = R_2\, I^2_{G_1\,\text{Min}}\, \frac{1{,}5\ \text{div}}{4\ \text{div}} = 75 \cdot 1^2\, \frac{1{,}5}{4} = 28{,}125\ \text{W}$$

$$\overline{P}_{\text{R3}} = R_3\, I^2_{G_1\,\text{Min}}\, \frac{1{,}5\ \text{div}}{4\ \text{div}} = 20 \cdot 1^2\, \frac{1{,}5}{4} = 7{,}5\ \text{W}$$

$$\overline{P}_{\text{R4}} = R_4\, I^2_{G_1\,\text{Max}}\, \frac{2{,}5\ \text{div}}{4\ \text{div}} = 25 \cdot 2{,}5^2\, \frac{2{,}5}{4} = 97{,}656\ \text{W}$$

$$\overline{P}_{\mathrm{R5}} = \frac{U_2^2}{R_5} = \frac{25^2}{50} = 12{,}5\ \mathrm{W}$$

$$\overline{P}_{\mathrm{Total\ R}} = 21{,}406 + 28{,}125 + 7{,}5 + 97{,}656 + 12{,}5 = 167{,}19\ \mathrm{W}$$

Alternativament,

$$\overline{P}_{\mathrm{Total\ R}} = \overline{P}_{\mathrm{G1}} - \overline{P}_{\mathrm{U2}}$$

$$\overline{P}_{\mathrm{G1}} = U_{G_{1\,\mathrm{Max}}}\, I_{G_{1\,\mathrm{Max}}}\, \frac{2{,}5}{4} + U_{G_{1\,\mathrm{Min}}}\, I_{G_{1\,\mathrm{Min}}}\, \frac{1{,}5}{4}$$

$$\overline{P}_{\mathrm{G1}} = 100 \cdot 2{,}5\, \frac{2{,}5}{4} + (-100)(-1)\, \frac{1{,}5}{4} = 193{,}75\ \mathrm{W}$$

$$I_{R_5} = \frac{U_2}{R_5} = \frac{25}{50} = 0{,}5\ \mathrm{A} \qquad I_{G_{1\,\mathrm{Max}}} = I_{U_2} + I_{R_5} \quad \rightarrow \quad I_{U_2} = I_{G_{1\,\mathrm{Max}}} - I_{R_5}$$

$$I_{U_2} = \begin{cases} 2{,}5 - 0{,}5 = 2\ \mathrm{A}\ \ (\text{durant } 2{,}5 \text{ div}) \\ -0{,}5\ \mathrm{A}\ \ (\text{durant } 1{,}5 \text{ div}) \end{cases}$$

$$\overline{P}_{\mathrm{U2}} = U_2\, I_{U_2} = 25 \left(2\, \frac{2{,}5}{4} - 0{,}5\, \frac{1{,}5}{4}\right) = 26{,}563\ \mathrm{W}$$

$$\overline{P}_{\mathrm{Total\ R}} = 193{,}75 - 26{,}563 = 167{,}19\ \mathrm{W}$$

Exercici 5

a)

$\boldsymbol{\Gamma = 0}$**,** doncs està en buit

b)

$$\boldsymbol{\eta(\%) = 100\ \frac{P_{\text{útil}}}{P_{\text{consumida}}} = 100\ \frac{0}{1500} = 0\ \%}$$

c)

$$\cos\varphi_0 = \frac{P_0}{S_0} = \frac{P_0}{\sqrt{3}\,U_\text{N}\,I_0} = \frac{1500}{\sqrt{3}\cdot 690\cdot 30} = \frac{1500}{35854} = 0{,}042$$

d)

$$p = 3$$

e)

$$Q_0 = \sqrt{S_0^2 - P_0^2} = \sqrt{35854^2 - 1500^2} = 35823\ \text{var}$$

Exercici 6

a)

$$S = \sqrt{P^2 + Q^2} \quad \rightarrow \quad Q = \sqrt{S^2 - P^2} = \sqrt{(U\,A_1)^2 - P^2}$$

$$Q = \sqrt{(200\cdot 6{,}54)^2 - 1265^2} = 332{,}6\ \text{var}$$

b)

$$W = R_1\,A_1^2 + R_2\,I_{R_2}^2 \quad \rightarrow \quad I_{R_2} = \sqrt{\frac{W - R_1\,A_1^2}{R_2}} = \sqrt{\frac{1265 - 20\cdot 6{,}54^2}{15}} = 5{,}23\ \text{A}$$

$$U_{R_2} = R_2\,I_2 = 15\cdot 5{,}23 = 78{,}45\ \text{V}$$

$$I_\text{C} = \frac{U_{R_2}}{X_\text{C}} = \frac{U_{R_2}}{\frac{1}{\omega\,C}} = \frac{78{,}45}{\frac{1}{2\,\pi\,50\cdot 160\cdot 10^{-6}}} = \frac{78{,}45}{19{,}89} = 3{,}94\ \text{A}$$

c)

$$Q_C = X_C\, I_C^2 = 19{,}89 \cdot 3{,}94^2 = 308{,}8 \text{ var}$$

$Q = Q_L - Q_C \quad \rightarrow \quad Q_L = Q + Q_C = 332{,}6 + 308{,}8 = 641{,}4 \text{ var}$

$Q_L = X_L\, A_1^2 \quad \rightarrow \quad X_L = \frac{Q_L}{A_1^2} = \frac{641{,}4}{6{,}54^2} = 15\ \Omega \quad \rightarrow \quad L = \frac{X_L}{\omega} = \frac{15}{2\ \pi\ 50} = 47{,}75 \text{ mH}$